Hartley and Colpitts Oscillator

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B.Sc Part-III

Paper-VI

Dr. Ayan Mukherjee,
Assistant Professor,
Department of Physics,
Ram Ratan Singh College, Mokama.
Patliputra University, Patna

Hartley Oscillator

Fig. shows the ckt. diagram of a Hartley Oscillator using jn. transistor in the CE mode. The resistor R_1, R_2 & R_E and the supply voltage V_{CC} establishes the dc operating pt. in the active region of the characteristics. The capacitor C_B is the blocking capacitor & C_E is an emitter bypass capacitor. Since the transistor operates in the CE mode it introduces a phase shift of 180° between its input and output voltages. The output voltage appears across the tank ckt. connected to the collector. A part of output voltage V_1 appearing across the inductance L_1 is the feedback voltage. The feedback voltage is 180° out of phase with the output voltage so that a net phase shift around the loop is 0° or 360°.

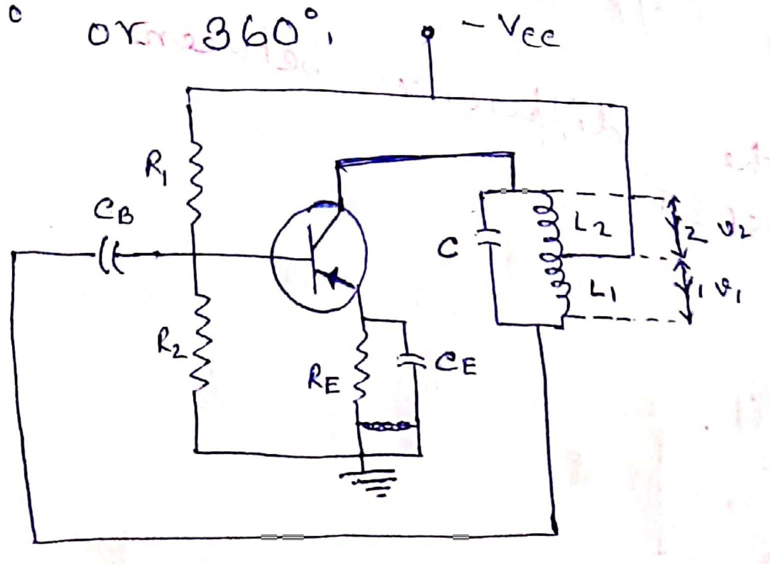

Fig-1.

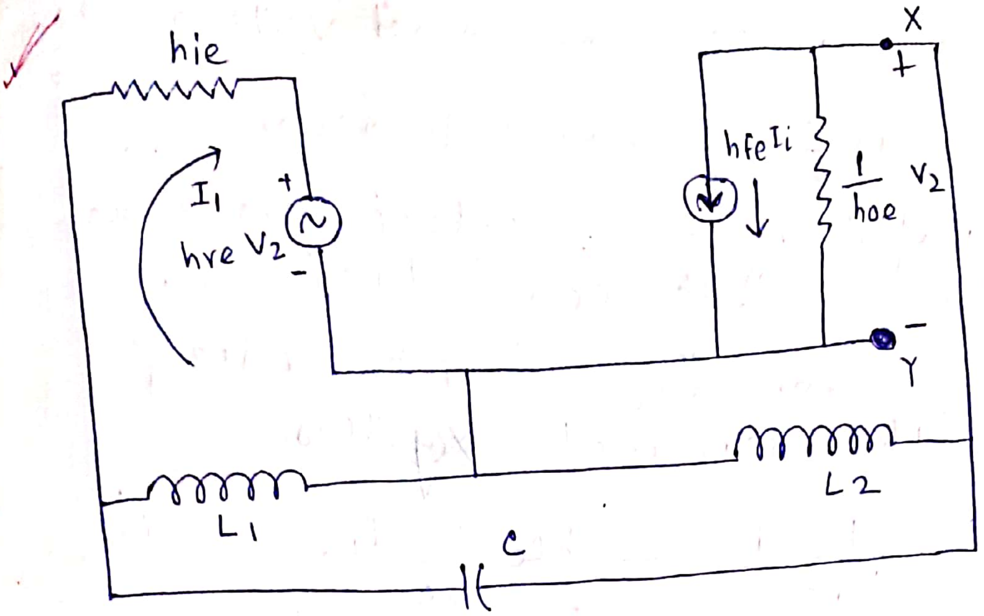

Fig → 2.

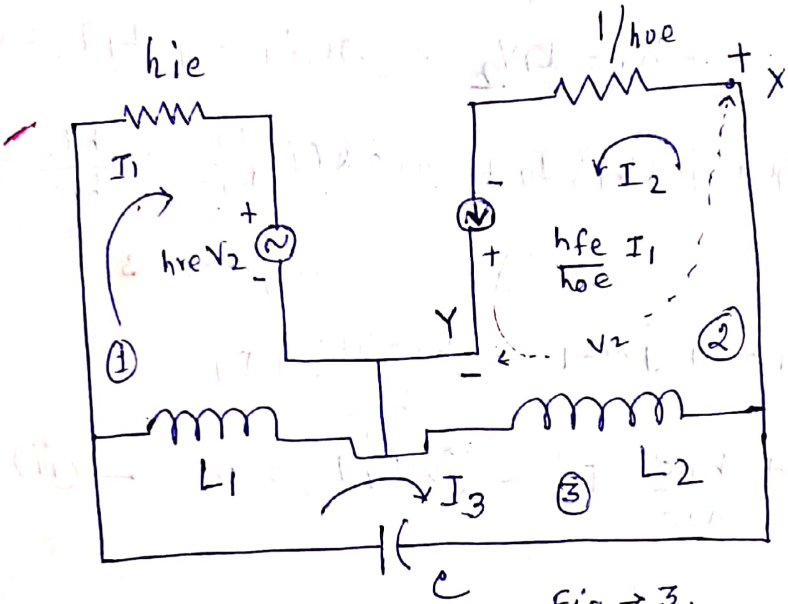

Fig → 3.

Analysis → Since the bias resistances $R_1, R_2, \& R_E$ are sufficiently large they will not affect the a c operating ckt. The hybrid model a.c. equivalent ckt of the heartley oscillator is shown in fig. Apply Thevnin's theorem ~~at~~ at the terminal and looking foward current source $h_{fe} I_1$ in left the current source $h_{fe} I_1$ in parallel with the resistance $\frac{1}{h_{oe}}$ can be removed by an equivalent ~~eq~~ thevnins eq voltage source of generated voltage $\frac{h_{fe}}{h_{oe}} I_1$

and internal impedence $\frac{1}{h_{oe}}$.

For simplicity we neglect the mutual inductance between L_1 & L_2.

From fig ③, the voltage across the terminal ~~xy~~ XY is

$$v_2 = \frac{1}{h_{oe}} I_2 - h_{fe} \frac{1}{h_{oe}} I_1 \quad \text{---- (i)}$$

Appling KVL for three loops.

For loop 1,

$$h_{ie} I_1 + h_{re} V_2 + j\omega L_1 I_1 - j\omega L_1 I_3 = 0$$

$$\left(h_{ie} + j\omega L_1\right) I_1 + h_{re} \left(\frac{I_2}{h_{oe}} - \frac{h_{fe}}{h_{oe}} I_1 \right) - j\omega L_1 I_3 = 0$$

$$\Rightarrow \left(h_{ie} + j\omega L_1 - \frac{h_{re} h_{fe}}{h_{oe}} \right) I_1 + \frac{h_{re}}{h_{oe}} I_2 - j\omega L_1 I_3 = 0 \quad \text{— (ii)}$$

For loop 2,

$$-\frac{h_{fe} I_1}{h_{oe}} + j\omega L_2 I_2 + j\omega L_2 I_3 + \frac{1}{h_{oe}} I_2 = 0 \quad \text{— (iii)}$$

Loop 3

$$j\omega (L_1 + L_2) I_3 - j\omega L_1 I_1 + j\omega L_2 I_2 + \frac{1 I_3}{j\omega C} = 0$$

$$\Rightarrow j\omega (L_1 + L_2) I_3 - j\omega L_1 I_1 + j\omega L_2 I_2 - \frac{j I_3}{\omega C} = 0 \quad \text{— (iv)}$$

ω is the angular freq. of oscillation

Since the currents I_1, I_2, I_3 are non-vanishing the determinant of the co-efficients of I_1, I_2 & I_3 must be zero. i.e.

$$\Delta = \begin{vmatrix} \left(h_{ie} + j\omega L_1 - \frac{h_{re} h_{fe}}{h_{oe}}\right) & \left(\frac{h_{re}}{h_{oe}}\right) & (-j\omega L_1) \\ -\frac{h_{fe}}{h_{oe}} & \left(j\omega L_2 + \frac{1}{h_{oe}}\right) & j\omega L_2 \\ -j\omega L_1 & j\omega L_2 & j\left(\omega L_1 + \omega L_2 - \frac{1}{\omega C}\right) \end{vmatrix} \quad \text{--(V)}$$

At the freq. of oscillation

$$j\omega L_1 + j\omega L_2 - \frac{j}{\omega C} = 0$$

$$\omega(L_1 + L_2) - \frac{1}{\omega C} = 0 \quad \text{———— (VI)}$$

$\Rightarrow$ ~~$\omega^2 = \frac{1}{}$~~

∴ Therefore eq^n (V), yields.

$$\left[h_{ie} + j\omega L_1 - \frac{h_{fe} h_{re}}{h_{oe}}\right] L_2^2 + \frac{h_{re}}{h_{oe}} L_1 L_2 - \frac{h_{fe}}{h_{oe}} L_1 L_2 + \left[\frac{1}{h_{oe}} + j\omega L_2\right] L_1^2 = 0 \quad \text{- - - - (vii)}$$

Equating the real part of eq^n (vii) to zero, we obtain.

$$\Delta_{he} L_2^2 - (h_{fe} - h_{re}) L_1 L_2 + L_1^2 = 0 \quad \text{—(viii)}$$

where $\Delta_{he} = h_{ie} h_{oe} - h_{fe} h_{re}$.

As $h_{re} \ll 1$

$$\Delta_{he} L_2^2 - h_{fe} L_1 L_2 + L_1^2 = 0 \quad \text{—(ix)}$$

Solving we gets,

$$L_2 = \frac{h_{fe}L_1 \pm \sqrt{h_{fe}^2 L_1^2 - 4\Delta_{he} L_1^2}}{2\Delta_{he}} \quad \text{— (10)}$$

Now, $h_{fe}^2 \gg 4\Delta_{he}$,

$$\therefore \quad L_2 \simeq \frac{h_{fe}}{\Delta_{he}} L_1 \quad \text{— (11)}$$

The eq^n (11) gives the condition for sustained oscillation.

Equating the imaginary part to zero, we obtain.

$$\frac{\omega}{c} L_1 L_2 + \frac{h_{ie}}{h_{oe}}\left(\omega L_1 + \omega L_2 - \frac{1}{\omega C}\right) = 0$$

or, $$\omega^2 = \frac{1}{\frac{h_{oe}}{h_{ie}} L_1 L_2 + C(L_1 + L_2)}$$

$\therefore$ The freq. of oscillation is

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi\left(\frac{h_{oe}}{h_{ie}} L_1 L_2 + CL_1 + CL_2\right)^{1/2}}$$

In practice, $\frac{h_{oe}}{h_{ie}} L_1 L_2 \ll C(L_1 + L_2)$

$$f \simeq \frac{1}{2\pi\left[C(L_1+L_2)\right]^{1/2}}$$

$$= \frac{1}{2\pi\sqrt{LC}}$$ [where $L = L_1 + L_2$ = Total inductance of the tank ckt]

Thus the ckt. gives oscillation at nearly the resonant freq. of the tank ckt.

Colpitts Oscillator :

Fig. shows the ckt. diagram of a Colpitts Oscillator using a transistor in CE mode. The resistors R_1, R_2, R_L & R_E and supply voltage Vcc established the dc operating point of the transistor in the active region of the characteristics. Since the transistor operates in CE mode it introduces a phase shift of 180° between its input and output voltages. ~~CB~~ The capacitor C_B blocks the dc current flow from the collector to the base ckt. through the coil of inductance L. C_E is an emitter bypass capacitor. The reactances of C_E & C_B are negligible at the freq. of oscillation. ~~The voltage across the capacitor~~

The fraction of output voltage appearing across the capacitor C_1 is the feedback voltage. The feedback voltage is 180° out of phase with the output voltage. So the net phase shift around the loop is 0° or 360°.

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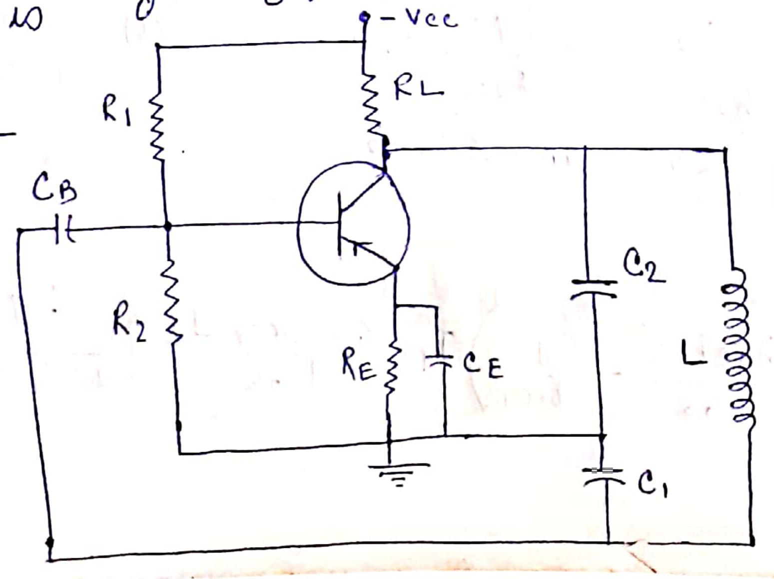
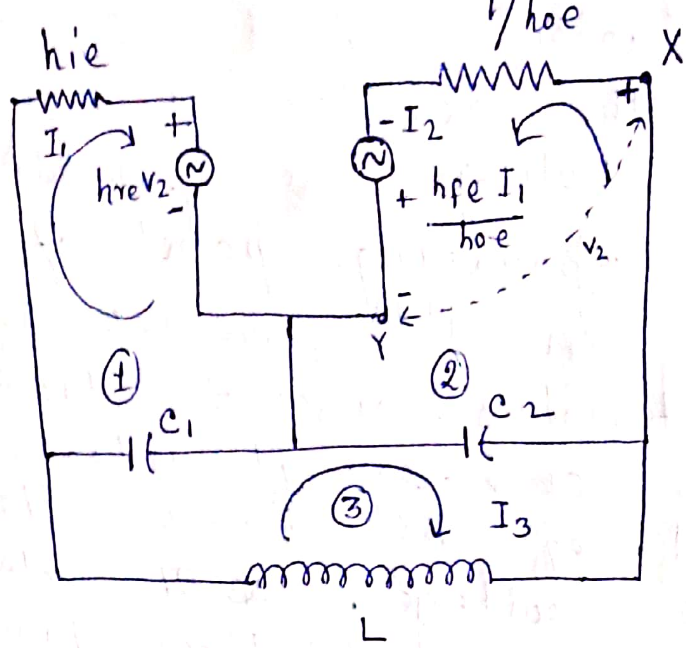


[ckt. diagram]

Ac equivalent ckt :→



Analysics → Since the resistor R_1, R_2, R_L & R_{EL} have sufficiently large values they do not affect the ac operation of the ckt. We neglect R_L as $R_L \gg \frac{1}{h_{oe}}$.

The potential difference between the terminal X & Y is

$$V_2 = \frac{I_2}{h_{oe}} - \frac{h_{fe}}{h_{oe}} I_1 \quad \ldots\ldots (i)$$

Applying KVL in 3 loops.

loop 1

$$\left(h_{ie} - \frac{h_{fe}h_{re}}{h_{oe}} - \frac{j}{wC_1}\right) I_1 + \frac{h_{re}}{h_{oe}} I_2 + \frac{j I_3}{wC_1} = 0 \quad -(ii)$$

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$$-\frac{h_{fe}}{h_{oe}} I_1 + \left(\frac{1}{h_{oe}} - \frac{j}{\omega C_2}\right) I_2 - \frac{j}{\omega C_2} I_3 = 0 \quad \text{(iii)}$$

$$\frac{1}{\omega C_1} I_1 - \frac{j I_2}{\omega C_2} + j\left(\omega L - \frac{1}{\omega C_1} - \frac{1}{\omega C_2}\right) I_3 = 0 \cdot \text{(iv)}$$

ω is the angular freq. of oscillation.

since the currents I_1, I_2, I_3 are non-vanishing the determinant of the co-efficients of I_1, I_2, I_3 must be zero. i.e.

$$\Delta = \begin{vmatrix} \left(h_{ie} - \frac{h_{fe}h_{re}}{h_{oe}} - \frac{j}{\omega C_1}\right) & \frac{h_{re}}{h_{oe}} & \frac{j}{\omega C_1} \\ -\frac{h_{fe}}{h_{oe}} & \left(\frac{1}{h_{oe}} - \frac{j}{\omega C_2}\right) & -\frac{j}{\omega C_2} \\ \frac{1}{\omega C_1} & -\frac{j}{\omega C_2} & j\left(\omega L - \frac{1}{\omega C_1} - \frac{1}{\omega C_2}\right) \end{vmatrix}.$$

At the freq. of oscillation,

$$j\left(\omega L - \frac{1}{\omega C_1} - \frac{1}{\omega C_2}\right) = 0. \quad \text{(v)}$$

$$\Delta h_e = h_{ie} h_{oe} - h_{fe} h_{re}.$$

$$\frac{C_1}{C_2} \approx \frac{h_{fe}}{\Delta h_e}.$$

$$\omega^2 = \frac{h_{oe}}{h_{ie} C_1 C_2} + \frac{1}{LC_1} + \frac{1}{LC_2}. \quad \text{(vi)}$$

The freq. of oscillation is

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\left(\frac{h_{oe}}{h_{ie} C_1 C_2} + \frac{1}{LC_1} + \frac{1}{LC_2}\right)^{1/2} \quad \text{(vii)}$$

Now, $h_{oe}/h_{ie} C_1 C_2 << \left[\frac{1}{LC_1} + \frac{1}{LC_2}\right]$, so eq^n (vii) reduces to,

$$f \simeq \frac{1}{2\pi\sqrt{LC_s}} \qquad \left[\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2}\right]$$